Some Introductory “Lessons” About Dissolved Gas Decompression Modeling

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General discussion:

First of all, I believe that before someone engages in the actual programming of a decompression model, they need to do quite a bit of research into the fundamentals behind that model. Usually this means the dissolved gas (Haldanian) model as implemented by Bühlmann and/or others. Unfortunately, the relevant information is not conveniently compiled or located in one handy reference. The books by Bühlmann have been the closest thing to an all-in-one reference, but the information is incomplete, especially if you want to program the model.

Bühlmann's work has to be taken in the historical context from which it was derived. Bühlmann did not "invent" most of the concepts that he presents in his books. He took the work done by others in the field before him and refined the model (slightly). The major elements of the dissolved gas model were developed by John S. Haldane, Robert D. Workman (U.S. Navy), and Heinz R. Schreiner (American researcher). Bühlmann relied heavily on the work of Robert Workman and communicated frequently with Schreiner as a colleague in the late 60's and early 70's. Bill Hamilton was a co-worker of Schreiner's in the early years.

Workman, Schreiner, and Bühlmann are deceased now. Bill Hamilton is still very active in the field and is probably one of the best ongoing sources for information on decompression topics. The key elements of the present day dissolved gas model, however, were laid down in a few research papers many years ago. These papers contain the core fundamentals about the model and its assumptions. This is information that every decompression programmer needs to read and to know. The references are listed as follows:

1. Boycott, A.E., Damant, G.C.C., & Haldane, J.S. "The Prevention of Compressed Air Illness," Journal of Hygiene, Volume 8, (1908), pp. 342-443. [This is the classic paper by Haldane and associates which started the field of decompression science. Haldane offers many insights (far ahead of his time) and a few misguided assumptions. There is a lot of talk about decompressing goats! It is well worth the time to read this paper. Much of it is still applicable today. An old 1908 copy of the Journal of Hygiene, Volume 8, can be found in the library of most major universities, especially those involved in medical science].

2. Workman, Robert D. "Calculation of Decompression Schedules for Nitrogen-Oxygen and Helium-Oxygen Dives," Research Report 6-65, U.S. Navy Experimental Diving Unit, Washington, D.C. (26 May 1965). [This paper is available through the National Technical Information Service (NTIS) or a photocopy can be ordered from the Undersea & Hyperbaric Medical Society (UHMS)].

The reason you should read the above papers is to understand the historical context and development of the dissolved gas model, which is necessary to understand Bühlmann's implementation of the model. Key points are as follows:

- **Haldane** established the concept of various "tissue" compartments within the body in which the gas loading behaves according to the law of exponential decay found throughout nature. Haldane also established the concept of "ascent limiting criteria," in his case it was through supersaturation ratios.

- **Workman** used the research data of the U.S. Navy to establish the concept of "M-values" for the ascent limiting criteria. These are expressed as a linear relationship between tolerated supersaturation in the "tissue" compartments and ambient pressure. Workman's M-values are based on the partial pressure of the inert gas in question, not on the total pressure of the breathing gas. Workman explained the concept that fast half-time compartments tolerate a greater supersaturation than slow half-time compartments. Workman also developed a detailed calculation procedure which is the foundation of those used today. A colleague of Workman's, William R. Braithwaite, later modified Workman's procedure to include the calculation of "tolerated ambient pressure" as a means to determine a "trial first stop."

- **Schreiner** explained the decompression model in terms of actual physiological elements such as gas transport in the blood to the tissues, solubility of gases in body fluids, fat fractions and composition of "tissue" compartments, and alveolar partial pressures of gases. He established the very important concept that the total inert gas partial pressure in a compartment is the sum of the partial pressures of all inert gases in that compartment, even if they have different half-times. Another major contribution that Schreiner made was to solve the differential equation for gas exchange when the ambient pressure changes at a constant rate. This is the general solution to the differential equation, of which the familiar instantaneous equation is only a subset. The general solution makes it possible to directly calculate the inert gas partial pressure of a compartment, as a function of time, for any linear (constant depth) or stepwise ascent or descent (at a constant rate). There are many other insights into decompression physiology given in this paper including a basis for the half-time constant, k.

As you can see from the above, many of the key elements used in the "Bühlmann Algorithm" were really developed by others and carried forward by Bühlmann. Of course, Bühlmann made a number of contributions to the science and practice of decompression calculations as well. His greatest contribution was to publish his book, in four editions from 1983 to 1995, as a nearly complete reference on making decompression calculations. Because this was the only "one-stop shopping" book widely available, it became the basis for most of the world's decompression computers and do-it-yourself programs. Key Bühlmann concepts, many explained in the 4th Edition (1995) of his book, are as follows:
The variation in half-times between two gases is inversely proportional to the square-roots of their molecular weights. This is a well-known relationship from chemistry called Graham's Law. It is particularly applicable when gases pass through a finely-pored membrane, a process called effusion which is a subset of diffusion.

The overpressure or supersaturation tolerance in a compartment is based upon the excess volume of gas tolerated by the body in that compartment. The tolerated partial pressures between two different gases in the same compartment will vary according to their solubilities in the transport medium that delivered those gases to that compartment (blood plasma, in this case).

The two key concepts above can be used to derive complete sets of half-times and M-values for other gases such as argon and neon (although due to their higher solubilities when compared to nitrogen and helium, respectively, they offer no substantial benefit for decompression under most scenarios).

The overall M-value for a compartment with multiple gases, each gas having different M-values, will vary in according to the proportion of each gas present in the compartment.

An explanation about Bühlmann "M-values" needs to be given. First of all, they are traditional M-values just as Workman defined them. Bühlmann simply modified the linear equation to suit his application. He started out with the traditional equation for an M-value in the form \( y = mx + b \) and he solved it for \( x \). This gives \( x = (y - b)/m \). To get rid of \( m \), the slope, in the denominator he just took the reciprocal and called it "Coefficient b." Traditional M-values are given as \( P = m(P_{amb}) + Mo \) [\( y = mx + b \) form], where \( P \) = tolerated inert gas partial pressure, \( m \) = slope, \( P_{amb} \) = ambient pressure, and \( Mo \) = intercept at sea level. Bühlmann expressed the same thing in absolute pressure coordinates. Bühlmann's Coefficient a is the intercept at \( P_{amb} = 0 \) and Bühlmann's Coefficient b is the reciprocal of the slope. It is easy to convert back and forth between Bühlmann M-values and traditional Workman-style M-values. This is something that I think a lot of people don't understand.

In the 1995 Edition of his book, Tauchmedizin or "Diving Medicine," Bühlmann gives a lot of insight into diving physiology and talks about much of his experience in the field over the years. He presents many of the results from his experimental research. In so doing, he also points out some of the shortcomings in the model. For example, he gives compartment partial pressures calculated at the end of dive series and expresses them in percent of the theoretical values. One thing is clear from his data. This is that, in every test series where incidences of DCS are shown, the affected divers are at a certain percentage less than the theoretical M-values in terms of compartment gas loading upon surfacing. This is usually in the range from 90% to 97%. The situation gets worse for repetitive dives which Bühlmann acknowledges and he cautions that reduction factors must be applied for repetitive diving calculations. One interpretation of Bühlmann's data is that his M-values do not represent a reliable line between NO SYMPTOMS and MASSIVE SYMPTOMS, but rather they represent a line between a LIMITED NUMBER OF SYMPTOMS and a MASSIVE NUMBER OF SYMPTOMS. This is consistent with most other decompression model experience which acknowledges that an M-value line is a solid line drawn through "a fuzzy gray area."

This kind of information should encourage decompression modelers who use the Bühlmann
M-values to incorporate an M-value reduction mechanism that is consistent across the entire ambient pressure range. One such mechanism is by reduction of the M-value Gradient. This is simply the difference between the M-value and ambient pressure.

Another good decompression reference, which I haven't mentioned previously, is Dr. Bruce Wienke. He has published several books which are available from Best Publishing Company. He is the author of the Reduced Gradient Bubble Model (RGBM). His excellent discussion about gradients is primarily focused on bubble models, but it is just as applicable to the dissolved gas model.

Well, I hope I have given you some encouragement to do some further reading into the fundamentals of decompression modeling. It is more than just programming a set of equations out of a book. There are a lot of complex considerations involved. I don't have all the answers, but collectively, we in the diving community can arrive at a lot of answers by exchanging information.

**Gas loading calculations and the Schreiner Equation:**

The fundamental relationship of the dissolved gas model is described by a differential equation:

\[
\frac{dP}{dt} = k(P_i - P)
\]

which states that the instantaneous rate of change of inert gas pressure \(\frac{dP}{dt}\) in a hypothetical tissue compartment is proportional to a time constant \(k\) multiplied by the gradient between the inspired inert gas pressure \(P_i\) and the present (or initial) compartment inert gas pressure \(P\). This kind of relationship is common in the natural sciences; Newton's Law of Cooling, for example. The main principle is that a GRADIENT is the driving force behind the rate of change in the amount of something.

In order to solve this differential equation for \(P\), compartment inert gas pressure, as a function of time, we must use integral calculus. Before we do this, however, there are two conditions we must consider. The first is when the inspired inert gas pressure, \(P_i\), remains constant such as during a constant depth dive profile. The second is when \(P_i\) changes with respect to time such as during ascents and descents. In order to simplify the integration in the second case, we will stipulate that the inspired inert gas pressure changes at a constant rate such as with a constant rate of ascent or descent, i.e. 10 msw/min.

Rather than writing out the integrals here, we will go right to the solutions! In the first case (constant depth), the solution is:

\[
P = P_0 + (P_i - P_0)(1 - e^{-kt})
\]

This is the "Haldane" equation or the "instantaneous" equation.

This same equation can also be written as:

\[
P = P_0 + (P_i - P_0)(1 - e^{(-ln2t/half-time)})
\]
(the latter form appears in Bühlmann's *Tauchmedizin* book). In the above equations:

- **P** = compartment inert gas pressure (final)
- **Po** = initial compartment inert gas pressure
- **Pi** = inspired compartment inert gas pressure
- **t** = time (of exposure or interval)
- **k** = time constant (in this case, half-time constant)
- **e** = base of natural logarithms
- **ln2** = natural logarithm of 2

In the second case, ascent or descent at a constant rate, the solution is:

\[
P = P_{io} + c(t - 1/k) - [P_{io} - P_{o} - (c/k)]e^{-kt}
\]

This is the general solution or "Schreiner" equation. It can also be written as:

\[
P = P_{io} + R(t - 1/k) - [P_{io} - P_{o} - (R/k)]e^{-kt}
\]

In the above equations:

- **P_{io}** = initial inspired (alveolar) inert gas pressure
- (**P_{io}** = initial ambient pressure minus water vapor pressure)
- **Po** = initial compartment inert gas pressure
- **c** = rate of change in inspired gas pressure with change in ambient pressure
  (this is simply rate of ascent/descent times the fraction of inert gas)
- **R** = same as **c**
- **t** = time (of exposure or interval)
- **k** = half-time constant = ln2/half-time (same as instantaneous equation)

Note that when **c** (or **R**) = 0 in the above equation, it is reduced to the more familiar instantaneous form, \( P = P_{o} + (P_{i} - P_{o})(1 - e^{-kt}) \).

**Examples of applying the gas loading equations:**

I would like to provide some advice about how to set up your program to calculate for ascent and descent portions of the dive profile. Many people use the instantaneous gas loading equation and "force" the computer to calculate the ascent or descent profile in greater resolution by dividing the segment into smaller and smaller increments. This is not an efficient way to calculate an ascent or descent profile!

The equation \( P = P_{o} + (P_{i} - P_{o})(1 - e^{-kt}) \) is applicable only for CONSTANT DEPTH profiles. This is a point that Bühlmann unfortunately does not explain in his book. To use this familiar equation for ascent or descent profiles requires you to break up the interval into very small increments to produce any kind of accuracy.
The efficient and direct calculation for gas loading during ascent and descent profiles is with the general solution given by Schreiner:

\[ P = P_{io} + R(t - 1/k) - [P_{io} - P_o - (R/k)]e^{-kt} \]

Where:

- \( P_{io} \) = initial inspired (alveolar) inert gas pressure
- \( P_o \) = initial compartment inert gas pressure
- \( R \) = rate of change in inspired gas pressure with change in ambient pressure
  
  (this is simply rate of ascent/descent times the fraction of inert gas)
- \( t \) = time
- \( k \) = half-time constant = \( \ln2/\text{half-time} \) (same as familiar equation)

Note that when \( R = 0 \) in the above equation, it is reduced to the more familiar form,

\[ P = P_o + (P_i - P_o)(1 - e^{-kt}) \]

The Schreiner equation is used to compute the partial pressure gas loading for each gas separately during ascent/descent. The sum of these is then the total compartment gas loading. The following examples are subroutines in FORTRAN (should be easy to follow!) from some of my programs which show the correct application of equations. This subroutine is for ascent/descent segments at a constant rate (i.e. descend to 100 fsw at 50 fsw/min.). Note that an ascent rate must be expressed as a negative number (i.e. -50 fsw/min).

```fortran
SUBROUTINE ASCDEC (SDEPTH, FDEPTH, RATE)
C
INTEGER MIXNUM, TEMPSG, SEGNUM
REAL FHE, FN2, KHE, KN2, PHEO, PN2O, PHE, PN2, PH2O
REAL FDEPTH, SDEPTH, PIHEO, PIN2O, RATE, RTIME, SGTIME, TEMPRT
REAL HERATE, N2RATE, SPAMB, FPAMB, PAMB
DIMENSION FHE (10), FN2(10), KHE(16), KN2(16)
DIMENSION PHEO(16), PN2O(16), PHE(16), PN2(16)
COMMON /A/ FHE, KHE, KN2, PH2O, /B/ RTIME, SEGNUM, FN2, SGTIME
COMMON /B/ MIXNUM, /C/ PHE, PN2, /D/ PAMB
SGTIME = (FDEPTH - SDEPTH)/RATE
TEMPRT = RTIME
RTIME = TEMPRT + SGTIME
TEMPSG = SEGNUM
SEGNUM = TEMPSG + 1
FPAMB = FDEPTH + 33.0
SPAMB = SDEPTH + 33.0
PAMB = FPAMB
PIHEO = (SPAMB - PH2O)*FHE(MIXNUM)
PIN2O = (SPAMB - PH2O)*FN2(MIXNUM)
HERATE = RATE*FHE(MIXNUM)
N2RATE = RATE*FN2(MIXNUM)
DO 430 I = 1,16
PHEO(I) = PHE(I)
PN2O(I) = PN2(I)
PHE(I) = PIHEO + HERATE*(SGTIME - 1.0/KHE(I)) -
* (PIHEO - PTHEO(I) - HERATE/KHE(I))*EXP (-KHE(I)*SGTIME)
PN2(I) = PIN2O + N2RATE*(SGTIME - 1.0/KN2(I)) -
* (PIN2O - PTN2O(I) - N2RATE/KN2(I))*EXP (-KN2(I)*SGTIME)
430 CONTINUE
```
This next subroutine is for constant depth segments (i.e. at 100 fsw for 15 minutes).

```fortran
SUBROUTINE CDEPTH (DEPTH, SRTIME)
C
INTEGER MIXNUM, TEMPSG, SEGNUM
REAL FHE, FN2, KHE, KN2, PHEO, PN2O, PHE, PN2, PH2O, SRTIME
REAL DEPTH, PAMB, PIHE, PIN2, RTIME, SGTIME, TEMPRT
DIMENSION FHE (10), FN2(10), KHE(16), KN2(16)
DIMENSION PHEO(16), PN2O(16), PHE(16), PN2(16)
COMMON /A/ FHE, KHE, KN2, PH2O, /B/ RTIME, SEGNUM, FN2, SGTIME
COMMON /B/ MIXNUM, /C/ PHE, PN2, /D/ PAMB
SGTIME = SRTIME - RTIME
TEMPRT = SRTIME
RTIME = TEMPRT
TEMPSG = SEGNUM
SEGNUM = TEMPSG + 1
PAMB = DEPTH + 33.0
PIHE = (PAMB - PH2O)*FHE(MIXNUM)
PIN2 = (PAMB - PH2O)*FN2(MIXNUM)
DO 520 I = 1,16
   PHEO(I) = PHE(I)
   PN2O(I) = PN2(I)
   PHE(I) = PHEO(I) + (PIHE - PHEO(I)) * 
            (1.0 - EXP (-KHE(I)*SGTIME))
   PN2(I) = PN2O(I) + (PIN2 - PN2O(I)) * 
            (1.0 - EXP (-KN2(I)*SGTIME))
520 CONTINUE
RETURN
END
```

With the two above subroutines (subprograms), a complete dive profile can be directly calculated in any combination of ascent/descent segments and constant depth segments (what some folks call waypoints).

**Example from a previous e-mail:**

... you mention that you use 3 second time slices to calculate with precision for the gas uptake/elimination during ascent and descent segments of the dive profile. This implies that the computer must calculate many iterations to arrive at a total for the interval. This process is unnecessary if you use the Schreiner equation which will directly make an exact calculation for the interval. If you are a student of calculus, you will recognize that the familiar instantaneous equation and the Schreiner equation are definite integrations of the same original differential equation. The difference is that with the instantaneous equation \[P = Po + (Pi - Po)(1 - e^{-kt})\], the inspired partial pressure of the gas, Pi, is assumed to be constant (i.e. as if you are staying at the same constant depth for the interval). The Schreiner equation, \[P = Po + R(t - 1/k) - [Pio - Po - (R/k)e^{-kt}]\], was integrated with the inspired partial pressure of the gas changing at a constant rate (such as during ascent or descent). This means that it gives you an exact number for the interval and there is no need to do "slices" or iterations. Let me give you an example:

Say you are descending from 0 to 120 fsw at 60 fsw/min and you're breathing trimix with 15% O2, 45% He, and 40% N2.
The time, \( t \), for this interval will be (Final Depth - Initial Depth)/Rate = (120 - 0)/60 = 2 min.

The initial ambient pressure, \( P_{amb} \), (in absolute coordinates) for this interval will be Depth + 33 = 0 + 33 = 33 fsw.

The initial inspired partial pressure for helium, \( P_{Heo} \), will be the initial ambient pressure, \( P_{amb} \), minus the water vapor pressure in the alveoli times the fraction of inert gas - \( P_{Heo} = (P_{amb} - PH2O)\cdot F_{He} \). The water vapor pressure is always considered to be constant in the alveoli. The value given by Buhlmann and converted to fsw is 2.042 fsw. Thus,
\[
P_{Heo} = (33 - 2.042)\cdot 0.45 = 13.93 \text{ fsw}.
\]

The initial inspired partial pressure for nitrogen, \( P_{N2o} \), will be the initial ambient pressure, \( P_{amb} \), minus the water vapor pressure in the alveoli times the fraction of inert gas -
\[
P_{N2o} = (P_{amb} - PH2O)\cdot F_{N2}.
\]
\[
P_{N2o} = (33 - 2.042)\cdot 0.40 = 12.38 \text{ fsw}.
\]

The rate of change of inspired helium pressure with time, \( R_{He} \), will be the descent rate, 60 fsw/min, times the fraction of inert gas -
\[
R_{He} = 60\cdot 0.45 = 27 \text{ fsw/min}.
\]

The rate of change of inspired nitrogen pressure with time, \( R_{N2} \), will be the descent rate, 60 fsw/min, times the fraction of inert gas -
\[
R_{N2} = 60\cdot 0.40 = 24 \text{ fsw/min}.
\]

Let's say this is the first dive of the day and you have not been diving recently. The initial partial pressure of helium, \( P_{Heo} \), in your compartments is zero. Thus \( P_{Heo} = 0.0 \).

The initial partial pressure of nitrogen, \( P_{N2o} \), in your compartments is "saturation" at sea level due to breathing atmospheric air, \( P_{N2o} = 33 \text{ fsw (absolute)} \) minus water vapor pressure times the fraction of inert gas,
\[
P_{N2o} = (33 - 2.042)\cdot 0.79 = 24.46 \text{ fsw}.
\]

For purposes of this example, let's just look at Buhlmann Compartment No. 1 for helium and nitrogen:

Helium half-time = 1.51 min, Nitrogen half-time = 4.0 min. The time constant for helium, \( k_{He} \), is the natural logarithm of 2 divided by the half-time -
\[
k_{He} = \ln 2/\text{half-time} = 0.693/1.51 = 0.459
\]

The time constant for nitrogen, \( k_{N2} \), is the natural logarithm of 2 divided by the half-time -
\[
k_{N2} = \ln 2/\text{half-time} = 0.693/4.0 = 0.173
\]

Therefore, to determine what the exact partial pressure for helium and nitrogen in Compartment No. 1 will be upon arrival at a depth of 120 fsw, use the Schreiner equation:

\[
P_{He} = P_{Heo} + R_{He}(t - 1/k_{He}) - [P_{Heo} - P_{Heo} - (R_{He}/k_{He})]e^{-k_{He}\cdot t}
\]
\[
P_{He} = 13.93 + 27(2 - 1/0.459) - [13.93 - 0 - (27/0.459)]e^{-0.459\cdot 2}
\]
\[
P_{He} = 27.03 \text{ fsw (absolute)}
\]
PN2 = PiN2o + RN2(t - 1/KN2) - [PiN2o - PN2o - (RN2/kN2)]e^-kN2*t
PN2 = 12.38 + 24(2 - 1/0.173) - [12.38 - 24.46 - (24/0.173)e^-0.173*2
PN2 = 28.36 fsw (absolute)

The total compartment inert gas pressure, PIG, will be the sum of PHe and PN2 -
PIG = 27.03 + 28.36 = 55.39 fsw (absolute)

Of course you will need to run the same calculations for each of the 16 compartments.

Now compare the method of this example with that of using the instantaneous equation, \( P = P_0 + (P_i - P_0)(1 - e^-kt) \). If you were to divide the interval from 0 to 120 fsw up into slices of 3 seconds each, it would require 40 iterations of the calculation to arrive at the same result. (2 min = 120 seconds divided by 3 second slices = 40 iterations). This means that for 16 compartments and 2 gases, the computer will have to do 40 * 16 * 2 = 1280 calculations. The result that you get will be an approximation to the actual gas loading (i.e. like the trapezoid rule of numerical integration from calculus). Using the Schreiner equation it will only require 32 calculations and produce the EXACT result!

If you do not believe me, then try writing a couple of little programs to test each method and see what you come up with.

Water vapor pressure, respiratory quotient, and alveolar gas adjustment:

COMPOSITION OF ALVEOLAR GAS

Alveolar gas does not have the same concentrations of gases as the breathing gasmix. There are several reasons for the differences. First the alveolar gas is only partially replaced by breathing gas with each breath. Second, oxygen is constantly being absorbed from the alveolar gas. Third, carbon dioxide is constantly diffusing from the pulmonary blood into the alveoli. And fourth, dry breathing gas that enters the respiratory passages is humidified even before it reaches the alveoli.

HUMIDIFICATION OF THE GAS AS IT ENTERS THE RESPIRATORY PASSAGES

Breathing gas normally contains almost no carbon dioxide and little water vapor. However, as soon as the breathing gas enters the respiratory passages, it is exposed to the fluids covering respiratory surfaces. Even before the gas enters the alveoli, it becomes totally humidified. The partial pressure of water vapor at normal body temperature of 37 deg C is 47 mm Hg, which, therefore is the partial pressure of water in the alveolar gas. Since the total pressure in the alveoli cannot rise to more than ambient pressure [without pulmonary barotrauma], this water vapor simply expands the volume of the gas and thereby DILUTES all other gases in the inspired gasmix.

The body maintains a constant water vapor pressure of 47 mm Hg in the alveoli at normal body temperature. Dry breathing gases are a contributing factor to dehydration when diving since the body must supply water to maintain this vapor pressure.
RESPIRATORY QUOTIENT (ALVEOLAR)

The ratio of carbon dioxide production to oxygen consumption is called the respiratory quotient. Depending on diet or physical exertion, this value ranges from 0.7 to 1.0. From U.S. Navy Diving Manual: 0.9 is a good rule-of-thumb value for making calculations.

FROM BENNETT & ELLIOTT (4TH EDITION):

Alveolar ventilation: the total 'dry gas' pressure in the alveoli, including water vapor pressure, will be essentially equal to barometric or ambient pressure. The water vapor pressure must be subtracted from this. It depends on the alveolar temperature and is 47 mm Hg (6.25 kPa) at 37 deg C.

FROM BÜHLMANN (1995 DIVING MEDICINE):

For calculation of the partial pressures of a breathing gas, the water vapor pressure must be subtracted from the total ambient pressure:

\[ P_i \text{ (inert gas)} = (P_{amb} - 0.627 \text{ bar}) \times \text{Fraction (inert gas)} \]

Without consideration of the water vapor pressure, a higher partial pressure is calculated for O2, N2, He, etc. In experimental decompression research, without this fact of respiratory physiology, the straightforward calculation of the partial pressures of breathing gases results in higher-than-actual values for the tolerated inert gas overpressure.

FROM HAMILTON DCAP MANUAL (1997):

Alveolar gas: the operational assumptions of DCAP are based on inspired gas values, but the Tonawanda II (dissolved gas model) uses alveolar levels for the computations. This is a major difference between DCAP and some other decompression computational programs.

Rationale for alveolar gas adjustment: inspired gas is diluted by CO2 and water vapor in the lungs. These are not particularly important in the pressures used in diving, but they can become significant in altitude calculations where they make up a greater fraction of the total pressure. At a respiratory quotient assumed to be 0.8, the Alveolar Ventilation Equation yields an alveolar inert gas pressure (see Schreiner and Kelley, 1971):

\[ P_{alveolar \text{ (inert gas)}} = (P_{amb} - 37 \text{ mm Hg}) \times \text{Fraction (inert gas)} \]

\[ 37 \text{ mm Hg} = 0.0493 \text{ bar} = 0.493 \text{ msw} \]
\[ 37 \text{ mm Hg} = 0.0487 \text{ atm} = 1.607 \text{ fsw} \]

The appropriate value is subtracted from ambient pressure during DCAP's calculation of alveolar gas.

SUMMARY FOR DECOMPRESSION PROGRAM CALCULATIONS:

As can be seen above, there is no question that water vapor pressure must be subtracted from
ambient pressure when calculating the partial pressure of an inspired (alveolar) breathing gas. The only question is what value to subtract. This question is put into perspective by use of the Alveolar Ventilation Equation:

\[ P_{alveolar} = \left( \frac{PCO_2 \times (1 - Rq)}{Rq} + P_{amb} - PH_{2O} \right) \times \text{Fraction (inert gas)} \]

where \( Rq \) = respiratory quotient, \( PCO_2 \) = carbon dioxide pressure, \( P_{amb} \) = ambient pressure, and \( PH_{2O} \) = water vapor pressure.

If we use standard values of \( PCO_2 = 40 \text{ mm Hg} \) and \( PH_{2O} = 47 \text{ mm Hg} \), then for the following respiratory quotients we get:

<table>
<thead>
<tr>
<th>( Rq )</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{amb} )</td>
<td>37 mm Hg</td>
<td>42.55 mm Hg</td>
<td>47 mm Hg</td>
</tr>
<tr>
<td>( P_{amb} )</td>
<td>0.0493 bar</td>
<td>0.0567 bar</td>
<td>0.0627 bar</td>
</tr>
<tr>
<td>( P_{amb} )</td>
<td>1.607 fsw</td>
<td>1.848 fsw</td>
<td>2.041 fsw</td>
</tr>
<tr>
<td>(Schreiner value)</td>
<td>(U.S. Navy value)</td>
<td>(Bühlmann value)</td>
<td></td>
</tr>
</tbody>
</table>

So, the decompression modeler must choose between these values with the Bühlmann value being the least conservative, the Schreiner value being the most conservative, and the U.S. Navy value in the middle!

**Pressure units:**

The units used to describe depth in diving are units of PRESSURE and not units of LENGTH! This is a common source of confusion and misunderstanding and the conversion between these units is often done incorrectly.

Two systems have evolved in diving regarding pressure units - the system of American usage and the system of European usage. Neither system is particularly "correct" and neither system conforms entirely to SI standards of usage.

In both systems, the pressure units for depth are defined ARBITRARILY and INDEPENDENTLY:

The American usage unit of depth (pressure) is feet of seawater (fsw) and is DEFINED as,

\[ 1 \text{ fsw} = \frac{1}{33} \text{ standard atmosphere} = 3.0705 \text{ kPa} = 3.0705 \times 10^3 \text{ Pascals (N/m}^2) \]

This unit conforms to a specific gravity for sea water of 1.020.

The European usage unit of depth (pressure) is the meter of seawater (msw) and is DEFINED as,

\[ 1 \text{ msw} = \frac{1}{10} \text{ bar} = 10 \text{ kPa} = 10^4 \text{ Pascals (N/m}^2) \]

This conforms to a specific gravity for sea water of 1.027.
Note that the units fsw and msw LOOK LIKE units of LENGTH but they are NOT!

The conversion between these PRESSURE UNITS is 3.2568 fsw/msw and 0.30705 msw/fsw.

The conversion between the linear units of LENGTH is 3.2808 feet/meter and 0.3048 meters/foot. These conversion factors are often INCORRECTLY applied to convert between fsw and msw.

Sea water ranges in specific gravity between about 1.020 and 1.030. Thus, both the American and European units were chosen to stay within these limits. The density of the water (or the depth for that matter) does not matter if the PRESSURE of the diver is measured and used to determine the proper decompression. Thus a dive in fresh water involves no adjustment as long as the diver's PRESSURE is measured with the same gauge calibration as is used for the decompression calculations.

Some important notes about USAGE CONVENTIONS need to be mentioned:

1. In the American system, sea level is considered to be at STANDARD ATMOSPHERIC PRESSURE = 101.325 kPa = 1.01325 bar = 760 mm Hg.

2. Some users of the American system, including the U.S. Navy and the Journal of Undersea and Hyperbaric Medicine, deviate from the standard definition of fsw and define 1 fsw = 1/33.08 standard atmosphere. However, most users of the American system define 1 fsw = 1/33 standard atmosphere or 33 fsw = 1 atm.

3. In the European system, sea level is considered to be 1.0 bar. The standard atmosphere, however, is 1.01325 bar. This means that the European system differs from the American system by 13.25 mb or 1.3%.

The daily fluctuations in barometric pressure will most likely outweigh the trivial differences between the American system and the European system.

It is important to note, however, that all calculations and measurements should be made in either one system or the other. Do not convert back and forth between the two systems. All calculations should be made against the same base and units should be kept consistent.

Another point worth mentioning is that neither the American unit, fsw, nor the European unit, bar, conform to SI standard usage. Technically, we should all be using the SI unit of pressure, the Pascal. However, the common practices of usage are so entrenched that this is unlikely to change.

Decompression programmers need to keep in mind the conventions of usage when setting up programs to operate in both fsw and msw. For the American system, fsw, sea level will be at 33 fsw = 1 atm. For the European system, msw, sea level will be at 10 msw = 1.0 bar.

Also, decompression programs should make all calculations in the pressure units used for depth. For example, gas loadings and M-values in the American system will all be in fsw. Gas loadings and M-values in the European system will all be in msw. This means that if the Bühlmann
M-values are used, for example, the "a" Coefficients need to be converted from bar to fsw or msw as the base data. In the msw case, its easy, just multiply by 10.

PROPER CONVERSION OF FEET OF SEAWATER (FSW) TO METERS OF SEAWATER (MSW)


DEFINITIONS
----------
1. One MSW is DEFINED as 1/10 bar
2. One FSW is DEFINED as 1/33 atm

Note: FSW and MSW are units of PRESSURE and NOT length!

SI CONVERSIONS
-------------

SI unit of pressure is the pascal (Pa) = Newtons per square meter

1 bar = 100 kPa
1 atm = 1.013250E+05 Pa (Atmosphere, standard)
1 atm = 1.01325 bar

CONVERSION OF UNITS
-------------------

\[
1 \text{ MSW} = (0.1 \text{ bar}) \times \frac{1 \text{ atm}}{1.01325 \text{ bar}} \times \frac{33 \text{ FSW}}{1 \text{ atm}} = 3.25684678 \text{ FSW}
\]

[continued next page]
# SPREADSHEET CALCULATIONS

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Follow-up:

As a follow-up to the previous material, I would like to make a few more points. The standard practice for decompression calculations is to calculate everything (applicable) in the pressure units of depth that you are using. This will be either fsw or msw. This post will show examples of what I am talking about.

First, however, a quick point about half-times. Bühlmann published a decompression model (ZH-L16) with sixteen (16) half-times for sixteen (16) compartments. There are actually two sets of values given for Compartment No. 1. Bühlmann calls these 1 and 1b. The difference is that Compartment No. 1 has half-times of 4.0 min N2/1.51 min He and Compartment No. 1b has half-times of 5.0 min N2/1.88 min He. These are NOT two different compartments - you must choose one or the other for Compartment No. 1. The reason that Bühlmann did this is that most other decompression modelers consider 5 min as the fastest half-time for nitrogen and by choosing the Compartment 1b values you will be consistent with the other models. The 4.0 min N2/1.51 min He half-times are too fast for most applications. They would really only be useful where extreme bounce dives are involved. Most technical diving profiles rarely involve the fastest compartment anyway.

Below is an example of initializing these half-times in a FORTRAN program. Note that I have chosen the 1b values for Compartment No. 1:

```
DATA HALFTH(1)/1.88/, HALFTH(2)/3.02/, HALFTH(3)/4.72/, *
  HALFTH(4)/6.99/, HALFTH(5)/10.21/, HALFTH(6)/14.48/, *
  HALFTH(7)/20.53/, HALFTH(8)/29.11/, HALFTH(9)/41.20/, *
  HALFTH(10)/55.19/, HALFTH(11)/70.69/, HALFTH(12)/90.34/, *
  HALFTH(13)/115.29/, HALFTH(14)/147.42/, HALFTH(15)/188.24/, *
  HALFTH(16)/240.03/
DATA HALFTN(1)/5.0/, HALFTN(2)/8.0/, HALFTN(3)/12.5/, *
  HALFTN(4)/18.5/, HALFTN(5)/27.0/, HALFTN(6)/38.3/, *
  HALFTN(7)/54.3/, HALFTN(8)/77.0/, HALFTN(9)/109.0/, *
  HALFTN(10)/146.0/, HALFTN(11)/187.0/, HALFTN(12)/239.0/, *
  HALFTN(13)/305.0/, HALFTN(14)/390.0/, HALFTN(15)/498.0/, *
  HALFTN(16)/635.0/
```

Next, is an example of initializing the value for water vapor pressure. This example uses the Bühlmann value (respiratory quotient = 1.0). For now, I suggest that you use this value since it will be consistent with the rest of the Bühlmann methodology. Please note that this value is expressed in msw, and NOT IN BAR, because it is standard practice to calculate everything in the pressure units of depth that we are using:

```
PH2O = 0.627  << expressed in msw !
```

Next, is an example of initializing some other variables (arrays) in FORTRAN. This program example will be for a first dive (not repetitive) and for diving at sea level. Note that the initial partial pressure for He will be 0.0 in all compartments. The initial partial pressure for N2 in all compartments will be saturation due to breathing atmospheric air AT SEA LEVEL. In the European system, this will be calculated as follows:

Sea level pressure = 1.0 bar = 10 msw
PN2 (saturation) = (Pamb - PH20) * Fraction (inert gas) = (10 msw - 0.627 msw) * 0.79 = 7.40467 msw

Again, note that we are expressing everything in msw because it is standard practice to calculate in the units of depth being used.

Programming example:

```plaintext
DO 25 I = 1,16
    CMPTMT(I) = I
    KHE(I) = ALOG(2.0)/HALPTH(I)
    KN2(I) = ALOG(2.0)/HALFTN(I)
    PHE(I) = 0.00
    PN2(I) = 7.40467
25 CONTINUE
```

Next is an example of making calculations in the ascent/descent subroutine. For the interval of ascent or descent, the starting ambient pressure (SPAMB) will be the starting depth (SDEPTH) plus the barometric pressure at sea level and the final ambient pressure will be the final depth (FDEPTH) plus barometric pressure at sea level. Note that ambient pressure is always the ABSOLUTE ambient pressure. Again, the pressures are expressed in msw and NOT in bar:

```plaintext
SPAMB = SDEPTH + 10.0  << msw !
FPAMB = FDEPTH + 10.0  << msw !
```

Similar example for constant depth subroutine:

```plaintext
PAMB = DEPTH + 10.0  << msw !
```

Also, in the constant depth subroutine is an example showing the inspired gas pressures being calculated in the same units as depth. For this example, assume you are at an absolute ambient pressure of 20 msw (2.0 bar) and your breathing trimix 17/33:

```plaintext
PIHE = (PAMB - PH2O)*FHE(MIXNUM)
PIN2 = (PAMB - PH2O)*FN2(MIXNUM)
```

Thus,

```plaintext
PIHE = (20 msw - 0.627 msw) * 0.33 = 6.393 msw
PIN2 = (20 msw - 0.627 msw) * 0.50 = 9.686 msw
```

Next is an important point to be made. In technical diving, the partial pressure of oxygen (PO2) is referenced in terms of atmospheres absolute (ATA or atm), especially when dealing with oxygen toxicity calculations. The scientific data and calculations for oxygen toxicity are all reported in units of atmospheres absolute. Therefore, in the European system, you MUST NOT calculate the partial pressure of oxygen (PO2) based on a sea level atmospheric pressure of 10 msw (1.0 bar) even though you do this for the gas loadings and other calculations. You must calculate the PO2 (in atm) based on the msw value for standard atmosphere (atm):

```plaintext
1 atm = 101.325 kPa = 1.01325 bar = 10.1325 msw
```
Thus, for the PO2 calculation (in atm) only:

\[ \text{PO2} = \left( \frac{\text{PAMB}}{10.1325} \right) \times \text{FO2} \]

Example using absolute ambient pressure of 20 msw (2.0 bar) and trimix 17/33:

\[ \text{PO2} = \left( \frac{20 \text{ msw}}{10.1325 \text{ msw}} \right) \times 0.17 = 0.335 \text{ atm} \]

This value of PO2 (atm) can then be used in oxygen toxicity calculations and the result will be consistent with the units in which the scientific data has been reported.

Also note that we DO NOT subtract water vapor pressure from the ambient pressure when calculating the PO2. This is in order to give a "worst case" value and to match PO2 calculations made by hand.

A final example to emphasize the standard procedure of performing decompression calculations in the units of depth being used. The Bühlmann M-value Coefficient \( a \), which is reported in units of bar, must be converted to a "base" unit in the depth units being used. [The Bühlmann Coefficient \( b \) does not require any conversion because it is dimensionless (the slope) and is the same for any pressure unit being used]. In the following example, the Bühlmann Coefficient \( a \) is converted to msw (multiply the bar value by 10) and becomes the base unit in a FORTRAN block data subprogram which initializes these variables in arrays in the program. Also note that in this case, the values in Compartment No. 1 are the Bühlmann 1b values for the 5.0 min N2/1.88 min He half-times:

```
BLOCK DATA COEFFS
REAL AHE, BHE, AN2, BN2
DIMENSION AHE(16), BHE(16), AN2(16), BN2(16)
COMMON /A/ AHE, BHE, AN2, BN2
DATA AHE(1)/16.189/, AHE(2)/13.830/, AHE(3)/11.919/, AHE(4)/10.458/, * AHE(5)/9.220/, AHE(6)/8.205/, AHE(7)/7.305/, AHE(8)/6.502/, * AHE(9)/5.950/, AHE(10)/5.545/, AHE(11)/5.333/, * AHE(12)/5.189/, AHE(13)/5.181/, AHE(14)/5.176/, * AHE(15)/5.172/, AHE(16)/5.119/
DATA BHE(1)/0.4770/, BHE(2)/0.5747/, BHE(3)/0.6527/, BHE(4)/0.7223/, * BHE(5)/0.7582/, BHE(6)/0.7957/, BHE(7)/0.8279/, BHE(8)/0.8553/, * BHE(9)/0.8757/, BHE(10)/0.8903/, BHE(11)/0.8997/, * BHE(12)/0.9073/, BHE(13)/0.9122/, BHE(14)/0.9171/, * BHE(15)/0.9217/, BHE(16)/0.9267/
DATA AN2(1)/11.696/, AN2(2)/10.000/, AN2(3)/8.618/, AN2(4)/7.562/, * AN2(5)/6.667/, AN2(6)/5.600/, AN2(7)/4.947/, AN2(8)/4.500/, * AN2(9)/4.187/, AN2(10)/3.798/, AN2(11)/3.497/, * AN2(12)/3.223/, AN2(13)/2.850/, AN2(14)/2.737/, * AN2(15)/2.523/, AN2(16)/2.327/
DATA BN2(1)/0.5578/, BN2(2)/0.6514/, BN2(3)/0.7222/, BN2(4)/0.7825/, * BN2(5)/0.8126/, BN2(6)/0.8434/, BN2(7)/0.8693/, BN2(8)/0.8910/, * BN2(9)/0.9092/, BN2(10)/0.9222/, BN2(11)/0.9319/, * BN2(12)/0.9403/, BN2(13)/0.9477/, BN2(14)/0.9544/, * BN2(15)/0.9602/, BN2(16)/0.9653/
END
```